Dynamic Computation Offloading in Ultra-Dense Networks Based on Mean Field Games

Renjun Zheng, Haibo Wang, *Member, IEEE*, Matthieu De Mari, *Member, IEEE*, Miao Cui, *Member, IEEE*, Xiaoli Chu, *Senior Member, IEEE*, and Tony Q. S. Quek, *Fellow, IEEE*

***Abstract*—In ultra-dense networks, the increasing popularity of computation intensive applications imposes challenges to the resource-constrained smart mobile devices (SMDs), which may be solved by offloading these computation tasks to the nearby mobile edge computing centers. However, when massive SMDs offload computation tasks in a dynamic wireless environment simultaneously, the joint optimization of their offloading decisions becomes prohibitively complex. In this paper, we firstly model the joint optimization problem as a multi-user non-cooperative dynamic stochastic game, then propose a mean field game based algorithm to solve it with a drastically reduced complexity. We derive the two partial differential equations ruling the optimal strategies of the mean field game, namely the Hamilton-Jacobi-Bellman and Fokker-Planck-Kolmogorov equations, which are solved in an iterative manner in our proposed algorithm. Numerical results demonstrate that the proposed mean field game-based offloading algorithm requires a lower cumulated cost than the conventional strategies under the latency constraints of computation tasks, with perfect prediction of future channel states. It also appears that the performance of the mean field game-based offloading strategy depends on the accuracy of the future channel knowledge provided to the system, as the uncertainty may compromise its cumulated cost performance.**

***Index Terms*—Computation offloading, mobile edge computing, ultra-dense network, mean field game.**

Manuscript received February 27, 2020; revised August 4, 2020, November 21, 2020, and February 4, 2021; accepted April 7, 2021. Date of publication April 29, 2021; date of current version October 11, 2021. This work was supported in part by the National Natural Science Foundation of China under Grant 61931001 and in part by the Science and Technology Plan Project of Guangdong Province under Grant 2019B010119001 and Grant 2021A0505030015. This article was presented in part at the Proceedings of the 2019 IEEE Globecom Workshops. The associate editor coordinating the review of this article and approving it for publication was G. Fodor. *(Corresponding author: Haibo Wang.)*

Renjun Zheng is with the China Aerospace Science and Technology Corporation, Beijing 100048, China (e-mail: 17120180@bjtu.edu.cn).

Haibo Wang is with the Research Institute of Broadband Wireless Mobile Communication, Beijing Jiaotong University, Beijing 100044, China (e-mail: hbwang@bjtu.edu.cn).

Matthieu De Mari and Tony Q. S. Quek are with the Information Systems Technology and Design Pillar, Singapore University of Technology and Design, Singapore 487372 (e-mail: matthieu_demari@sutd.edu.sg; tonyquek@sutd.edu.sg).

Miao Cui is with the School of Information Engineering, Guangdong University of Technology, Guangzhou 510006, China (e-mail: cuimiao@gdut.edu.cn).

Xiaoli Chu is with the Department of Electronic and Electrical Engineering, The University of Sheffield, Sheffield S1 3JD, U.K. (e-mail: x.chu@sheffield.ac.uk).

Color versions of one or more figures in this article are available at https://doi.org/10.1109/TWC.2021.3075028.

Digital Object Identifier 10.1109/TWC.2021.3075028

I. Introduction

THE explosive growth in the number of smart mobile devices (SMDs) accessing wireless networks is imposing high traffic demands on 5G networks [2]. Meanwhile, many emerging mobile applications like face recognition, natural language processing, virtual/augmented reality (VR/AR), network-assisted auto-driving and drone control, are both computation-intensive and energy-consuming, thus their performance might be compromised by the limited computation capacity and battery energy level in SMDs [3]. To address the traffic demand issue, ultra-dense networks (UDNs) can be deployed with a high density of small base stations (SBSs) or remote radio units (RRUs) sharing the same baseband units (BBUs) pool [4]. To solve the computation capacity and energy problem, the SMDs can offload their computation tasks to the nearby mobile edge computing (MEC) centers via the wireless links to their associated SBS/RRU. The MEC centers are usually equipped with CPU and GPU pools far more powerful than the SMDs and with fixed power supply that makes them much less sensitive to the energy consumption of computation tasks [5], [6]. However, when massive SMDs offload computation tasks to a MEC center in a dynamic wireless environment simultaneously, the joint optimization of their offloading decisions becomes prohibitively complex. In this paper, we investigate a dynamic computation offloading strategy with distributed decisions on SMDs in a UDN scenario.

A. Related Work and Motivation

In computation offloading, a SMD may offload a computation-intensive task, or a portion of it, to the MEC network instead of performing the task locally. In order to make the computation offloading decision, various aspects must be considered, e.g., the latency constraints of the computation applications, offloading energy cost, etc. In [7], the authors proposed partial offloading and binary offloading to achieve energy-efficiency. In order to minimize the overall energy consumption in a multi-task MEC system, Dai *et al.* [8] proposed an efficient computation offloading algorithm by jointly optimizing user association and computation offloading, where both computation resource allocation and transmission power allocation were considered. In ultra-dense Internet-of-Things (IoT) networks, an iterative searching-based task offloading scheme was proposed by Guo *et al.* [9], which jointly optimized task offloading, computational frequency scaling, and transmit power allocation. The aforementioned computation offloading schemes were obtained by solving centralized optimization problems, which require global network information at the cost of significant signaling overhead, e.g., signals to report the channel and queuing states of all the SMDs to the MECs. In addition, the centralized offloading optimization problems are usually complex, and may have to be solved using heuristic algorithms (which are sub-optimal). Moreover, since the SMDs are likely to be owned by different individuals with potentially different preferences of service, they may be reluctant to follow the centralized solution that maximizes the system performance rather than their individual utilities. All these make the design of an efficient distributed multi-user computation offloading mechanism attractive.

In distributed computation offloading strategies, multiple SMDs are making offloading decisions individually while individual competing for both radio resources and computation resources, which is often investigated with game theory [10]–[12]. In [10], the authors formulated the multi-user computation offloading problem as a potential game, and designed a distributed computation offloading algorithm which can achieve a Nash equilibrium in a multi-channel wireless contention environment. The authors of [11] proposed a distributed computation offloading scheme based on Stackelberg game, where the users compete for limited computation resources on edge clouds via a pricing approach. Cao *et al.* [12] formulated the multi-user computation offloading problem as a non-cooperative game and designed a machine learning-based computation offloading algorithm. From the game theory perspective, each SMD needs to consider all the other SMDs' possible decisions while making its own offloading decisions; otherwise, if too many SMDs choose to offload with a very high rate simultaneously (which leads to high transmission power for every SMD), then the severe interference among SMDs will degrade their achievable transmission rate, while the computation energy cost and/or delay at the MEC will increase dramatically, thus reducing the spectrum and energy efficiency. Most of the Game theory-based mechanisms solve this problem by letting each SMD take turns to update their decisions, where each SMD should watch its previous SMD's current decision, and then make its own decision. This process will keep repeating until a Nash equilibrium (NE) is reached. It can be deduced that, for a distributed offloading game with N SMDs, the convergence time will scale up as N increases, and finally reaches infinity when N is sufficiently large, whereas the channel condition may change before the algorithm converges. Such a drawback makes the traditional Game-based distributed offloading approach practically impossible to be adopted for UDN scenarios with a large number of SMDs.

Moreover, most existing works on computation offloading considered semi-static channels only, where the channel states, the offloading strategy and the resource allocation stay unchanged during a computation offloading session. The assumption of semi-static channels may limit the usefulness of the obtained results, since the transmission of computation data may not always be completed within the channel coherence time. Some recent works have demonstrated that future user mobility and channel variations could be accurately predicted [14], [15], and the wireless transmission strategies can be adapted to the predictable channel variations based on Dynamic Programming (DP) [16]. Furthermore, when multiple SMDs need to dynamically optimize their decisions in competing for limited communication and/or computation resources in distributed manner, the resulting problems become dynamic stochastic games (DSGs) [17]–[19]. By adopting DSG in distributed computation offloading problems, it is possible to adjust the SMDs offloading decisions both to the fast changing wireless environment, and to the other SMDs decisions, which shall bring great practical value to the distributed computation offloading study. However, a DSG with N players involves solving N correlated stochastic differential equations (SDEs), when N is large (as in UDNs), the solving process could become prohibitive complex to implement.

To tackle the scaling problem, mean field games (MFGs) were introduced for multi-player non-cooperative DSGs by Lasry and Lions in [20], which simplifies the interactions among numerous players into a two-body interaction by considering the players are equally rational. Applications of MFG in communication networks include power control in ultra-dense networks [21]–[23], interference management in device-to-device (D2D) communications [24], security enhancements in mobile ad-hoc networks [25], and computation offloading strategies in [26]–[28]. In [21]–[24], basically the mean field is employed to model the cumulative interference among all users, while each user need to decide its transmission power dynamically. A mean-field-type game was proposed in [26] for downlink computation offloading, where the distributed edge computing nodes decide how much they can offload from a task aggregator. In [27] and [28], the authors investigated the computation offloading strategy in the up-link using MFG approach. The authors of [49] adopted MFG to solve the resource allocation problem for uplink computation offloading to MEC in a non-orthogonal multiple access system. However, the work in [27], [28], [49] all formed the mean field using uplink interferences similar to [21]–[24], and didn't consider the competition for computation resources at the MEC.

Inspired by the application of MFG in communication networks, especially its great potential in reducing the complexity of solving a large number of DSGs as well as the scaling problem of traditional game-theoretical approaches, we investigate the MFG approach for the uplink distributed computation offloading problem in UDNs while considering the competition for computation resources and the dynamics of wireless channels.

B. Contribution of This Paper

In this paper, we aim at solving the dynamic uplink computation offloading decision problem with a distributed strategy, leading to an efficient utilization of both the SMDs' transmission energy and the MEC's computation resource. We consider the UDN scenario where a large number of SMDs are connected to a Fog-RAN, and share the computation resources in the MEC pool of this Fog-RAN [9], [31], [32]. Specifically, we propose an MFG-based distributed strategy to optimize each SMD's computation offloading decisions to minimize its total transmission energy and computation cost paid to the MEC jointly, where each SMD is able to adjust the offloading rate dynamically both to the varying channel conditions, and to the demand/competition for MEC computation resource by other SMDs. The contributions of this work are summarized as follows:

1) *A Mean Field Game Problem Formulation for Uplink Computation Offloading in UDNs:* We formulate a cost minimization problem for uplink computation offloading in UDNs by adopting the MFG theory, where the number of SMDs may grow to infinity. To the best of our knowledge, this work is the first to apply MFG to uplink computation offloading problems, while the only existing work on MFG-based computation offloading [26] studied the downlink computation offloading from a central task aggregator to distributed MEC nodes.
2) *Dynamic Uplink Computation Offloading Adapted to Both the Time-Varying Channel Condition and the Competition for MEC Computation Resource by Multiple SMDs:* In the formulated cost minimization problem, the cost function combines the transmission energy of each SMD and the computation price it has to pay to the MEC, where the latter is designed to increase with the normalized computation load of the MEC. Different from most of the existing works [15], [16], [21]–[24], [27], [28], [44] and [51] that adopted MFG to deal with the mutual interference among co-channel transmitters only, we apply MFG to also cope with the competition for shared MEC computation resource among a large number of SMDs through the design of a dynamic computation price.
3) *Distributed Mean Field Equilibrium Rate Offloading Strategy:* We derive the Hamilton-Jacobi-Bellman (HJB) equation and the Fokker-Planck-Kolmogorov (FPK) equation for the constructed MFG problem to achieve the Mean Field Equilibrium, which reveals the optimal offloading rate decisions at all time slots during the whole offloading time period for a generic SMD, and devise numerical approaches to solve these two coupled partial differential equations (PDEs). Based on the obtained solution, we propose a distributed mean field equilibrium rate offloading strategy (MFEROS) that can be implemented by each SMD to transmit more of its task bits to the MEC when its channel condition is better and/or the computation price is lower based on its prediction of the channel variation and the offloading decisions of other SMDs, while satisfying the latency constraint.
4) *The Proposed MFEROS Achieves a "Latency Gain" and a "Future Channel Knowledge Gain" Over the Existing Schemes:* Based on extensive simulation results, we reveal that the proposed MFEROS allows an SMD to exploit the entire time within the latency constraint for computation offloading by selecting the time slots when the channel condition is good and/or few other SMDs are offloading to offload its task bits, thus resulting in a "latency gain" over the offloading strategies that tend to complete offloading transmission as soon as possible. In the meantime, the proposed MFEROS enables an SMD to adapt its offloading transmission rate at every time slot to the present and the predicted future link quality, thus offering a "future channel knowledge gain" over the offloading strategies that adopt a constant transmission rate. The two gains lead to a much reduced cumulated computation offloading cost for SMDs than the exiting schemes.

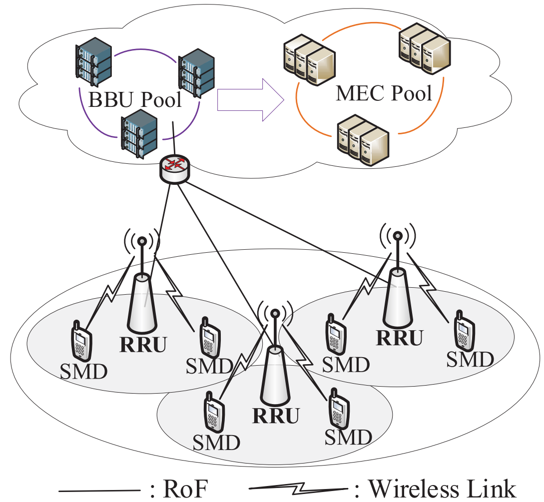


Fig. 1. System model.

The remainder of this paper is organized as follows. In Section II, we describe the system model under investigation. We construct the distributed optimization problem of uplink computation offloading as a multi-user non-cooperative DSG in Section III, and then transform the DSG into an equivalent MFG in Section IV. In Section V, numerical results are presented and discussed. Section VI concludes the paper.

II. System Model

As shown in Fig. 1, we consider a group of N SMDs in an UDN system which consists of L RRUs, and each SMD i has a computation-intensive task to be completed within time duration T. Each RRU serves at most X SMDs, and $N \leq L \times X$. Similar to the system architecture in [9], [31], [32], these RRUs connect to BBU pool through radio-over-fiber (RoF) links, and a MEC resource pool is co-located with the BBU pool. SMD i $(i \in \{1, \ldots, N\})$ may offload its computation tasks to this MEC pool through its nearest RRU, and the network operator charges the SMD for the offered computation offloading service. Thus, each SMD attempts to optimize its offloading decisions to minimize its total cost, including both transmission energy and computation price, within the time constraint T. It is worth noting that the RoF backhaul links might still be a bottleneck to a UDN [33], [34]. As the impact of non-ideal backhaul on UDN is outside the scope of this paper, for analytical tractability, we assume that the bandwidth of the RoF backhaul link for each RRU is sufficiently large, and thus the transmission delay between the BBU pool and the RRU can be neglected.

A. Transmission Model

Let $h_i(t)$ denote the channel gain between SMD i and its nearest RRU at time t. We model the channel dynamics as an Itô process [37], as in [21], [40], [42], i.e.,

$$dh_i(t) = \alpha_i\left(t, h_i(t)\right)dt + \sigma_i(t)dW(t). \tag{1}$$

where $\alpha_i\left(t, h_i(t)\right)$ is a deterministic smooth function, $\sigma_i(t)dW(t)$ is a Wiener processes following $\mathcal{N}(0, \sigma_i(t)dt)$, and the initial channel realization $h_i(0)$ is known. The deterministic function $\alpha_i\left(t, h_i(t)\right)$ models the path loss evolution over time due to user mobility, whereas the stochastic part $\sigma_i(t)dW(t)$ accounts for unpredictable channel variations, due to e.g., fast fading, or prediction uncertainty on the mobility of users.

The computation task of SMD i is to be served upon arrival at $t = 0$, and consists of $Q_i(0) = D > 0$ bits of data to be offloaded over the time duration T. Similar to [4], we assume the computation task can be split randomly, and different parts of it can be processed in parallel. Let $Q_i(t)$ denote the remaining data size which still needs to be offloaded at time t, and each SMD should decide its instantaneous offloading transmission rate $r_i(t)$. More specifically, the remaining data size decreases following the transmitted (offloaded) data volume, i.e.,

$$dQ_i\left(t\right) = -r_i\left(t\right)dt. \tag{2}$$

The computation task offloading is successful if all the data has been offloaded in time, i.e.,

$$Q_i(T) = 0, \tag{3}$$

where T is the offloading deadline required by the application-level quality of service (QoS). We assume that each time duration T is a resource allocation/scheduling period for computation offloading in a synchronized manner, while the asynchronous computation offloading scenarios will be investigated in our future work.

We assume that the SMDs access the same RRU via orthogonal frequency division multiple access (OFDMA), and fractional frequency reuse is employed to avoid the uplink interference among neighboring RRUs [29], thus the uplink intra-cell and inter-cell interference is mitigated in the investigated UDN system. However, it will be considered in our future work. Then, the achievable offloading transmission rate can be calculated as:

$$r_i\left(t\right) = B\log_2\left(1 + \frac{p_i\left(t\right)h_i\left(t\right)}{\sigma_n^2}\right), \tag{4}$$

where B stands for the channel bandwidth allocated to each SMD, $p_i\left(t\right)$ is the transmission power of SMD i at time t, and σ_n^2 denotes the power of the additive white Gaussian noise (AWGN). For practical reasons, we assume that the transmission power is bounded, i.e., $p_i(t) \in [0, p_i^{\max}]$, and therefore, the offloading rate is bounded as well, i.e.,

$$r_i\left(t\right) \in [0, r_i^{\max}\left(t\right)] = \left[0, B\log_2\left(1 + \frac{p_i^{\max}h_i\left(t\right)}{\sigma_n^2}\right)\right]. \tag{5}$$

Given the instantaneous offloading decision $r_i(t)$ and based on (4), we define the transmission cost of SMD i at t as:

$$C_i^{(1)}(t) = \eta p_i\left(t\right) = \eta\left(2^{\frac{r_i(t)}{B}} - 1\right)\frac{\sigma_n^2}{h_i\left(t\right)}, \tag{6}$$

where η is the coefficient to convert the value of power to monetary value. Therefore, the power cost expressed in Watts can be converted into the cost expressed in monetary units.

B. Computation Model

When the offloading task of SMD i is transmitted to the associated RRU, it can be assigned to any computation resource within the MEC pool thanks to the computation virtualization techniques [30]. Considering that the MEC pool has sufficient processing power [31], [32], we assume that each part of the offloaded tasks can be processed completely right after it is received. In the MEC pool, each task of an SMD can be assigned to a virtual machine with a specific CPU processing frequency by using computation virtualization techniques. However, due to the large number of SMDs in an UDN, it is impossible to assign a physical CPU or CPU processing core to each SMD's task, thus they will share the physical CPUs of the MEC pool. Although contemporary CPUs can adapt their working frequency to the computation load, the power (energy) cost of each physical CPU will increase twice as fast as its working frequency rises [35]. Hence, when multiple SMDs' tasks are sharing the same physical CPU, their accumulated offloading data volume will decide the working frequency of the serving CPU at the MEC, which incurs a quadratic cost of the total load of this CPU [26]. By considering the above cost model, and inspired by the auction model in [36], we define the computation price for each SMD i according to the MEC pool's normalized load, such as,

$$\Psi_i(t) = \begin{cases} \lambda, & N = 1 \\ \lambda + \dfrac{\xi}{N-1}\displaystyle\sum_{j=1, j\neq i}^{N} r_j(t), & N \geq 2, \end{cases} \tag{7}$$

where λ is the basic price for the MEC pool to process per unit data, and ξ is employed to convert the average load to monetary value. Then, the computation cost of SMD i at t can be expressed as:

$$C_i^{(2)}\left(t\right) = \Psi_i\left(t\right)r_i\left(t\right). \tag{8}$$

Based on this computation cost model, each SMD needs to consider all the other SMDs' possible decisions while making its own offloading decision; otherwise, if too many SMDs choose to offload with a very high rate simultaneously, the computation energy cost at the MEC will increase dramatically, resulting in a very low cost efficiency. That is, the price setting for $N \geq 2$ intends to prevent the SMDs from offloading with a very high rate simultaneously, thus the MEC's load will be distributed more evenly over time, which is more cost efficient for the SMDs and more energy efficient for the MEC.

Hence, the total cost for SMD i at t is given by

$$C_i\left(t\right) = C_i^{(1)}\left(t\right) + C_i^{(2)}\left(t\right). \tag{9}$$

III. Formulation of Multi-Player Dynamic Stochastic Game

In the investigated UDN system, SMD i $(i \in \{1,\ldots,N\})$ needs to decide the optimal instantaneous offloading strategy $\mathbf{r}_i^*(t)$, in a bounded action set $[0, r_i^{max}]$, which allows it to finish offloading its overall task data of size $Q_i(0)$ before the deadline T (a.k.a., the *delay constraint*), at a minimal cost. The optimal offloading decision $r_i^*(t)$ should be based on the present channel realizations, the remaining data size, and the prediction $(\alpha_i(t), \sigma_i(t))$ of future channel states (For simplicity, we assume $\alpha_i(t) = \alpha(t), \sigma_i(t) = \sigma_b, \forall i \in \{1,\ldots,N\}$.). Without loss of generality, the computation offloading optimization problem for each SMD can be formulated as a dynamic stochastic game (DSG) in continuous time as follow:

$$
\begin{aligned}
\mathbf{r}_i^* = \arg\min_{\mathbf{r}_i} & \,\mathbb{E}\left[\int_0^T C_i(t)dt\right], \\
s.t.\quad & \\
C1: \quad & dh_i(t) = \alpha_i\left(t, h_i(t)\right)dt + \sigma_i(t)dW(t), \\
C2: \quad & dQ_i(t) = -r_i\left(t\right)dt, \\
C3: \quad & Q_i(0) = D, \\
C4: \quad & Q_i(T) = 0.
\end{aligned} \tag{10}
$$

in which $C1$, $C2$ are the continuous time equivalent of Eq.(1) and (3), which describe the evolution of the channel state and the remaining data size of SMD i, and $C3$, $C4$ are the initial and the required final states of the data queue at each SMD, respectively. It is assumed that the initial channel state $h_i(0)$ and the stochastic differential equations (SDE) parameters $(\alpha(t), \sigma_b)$ are known at $t = 0$. Each SMD i attempts to solve its own version of the optimization problem (10) at the same time, leading to an N-user non-cooperative DSG. It is worth noting that the objective function in (10) and the constraints are smooth functions and first-order SDEs. This is later exploited to simplify the analysis of the DSG.

According to the dynamic programming theory [17], the optimal solution of (10) for the entire duration $[0, T]$ can be derived by constructing a running cost function (a.k.a, the Bellman function) over time period $[t, T]$, and then solved in a time-reversed order. Therefore, we define the running cost for SMD i $(i \in \{1,\ldots,N\})$ as follows:

$$
v_i\left(t, S_i\left(t\right)\right) = \min_{r_i(t)} \mathbb{E}\left[\int_{u=t}^T C_i\left(u\right)du + F\left(Q_i\left(T\right)\right)\right], \tag{11}
$$

where

$$
S_i\left(t\right) = \left[Q_i\left(t\right), h_i\left(t\right)\right] \tag{12}
$$

is the state of SMD i at current time t, which consists of the remaining data size $Q_i\left(t\right)$ and the channel state $h_i(t)$. It should be noted that in (11), $u \in [t, T]$ denotes the upcoming time from current time t to the end of this task. The Bellman function models the future cost on all upcoming time for SMD i, at time t and state $S_i(t)$. The offloading strategy to be used for all the SMDs are $\mathbf{r} = \left(r_i^*\left(u\right)\right)_{i\in\mathcal{N}, u\in[t,T]}$. $F\left(Q_i\left(T\right)\right)$ is a penalty function to relax the offloading completion constraint $C4$. Here, $F(Q_i(T)) = 0$, if $Q_i(T) = 0$. If $Q_i(T) > 0$, $F(Q_i(T))$ takes a sufficiently large positive value to punish the SMD for not fulfilling the offloading task in time.

Definition 1: A computation offloading strategy $\mathbf{R}^*\left(t\right) = \left(r_1^*\left(t\right), \ldots, r_N^*\left(t\right)\right)$ is a Nash equilibrium for the DSG in (10) if and only if $r_i^*\left(t\right)$ is the optimal control to the problem, i.e.,

$$
r_i^*(t) = \arg\min_{r_i^*(t)} \mathbb{E}\left[\int_t^T C_i\left(u, r_i\left(u\right), r_{-i}^*\right)du + F\left(Q_i\left(T\right)\right)\right], \tag{13}
$$

where r_{-i}^* denotes the offloading strategies of all the SMDs except for SMD i. Under the Nash equilibrium configuration, no SMD can have a lower cost by unilaterally deviating from its current offloading control policy.

According to [38], [39], a sufficient condition for the existence of a Nash equilibrium is the existence of N inter-dependent solutions $v_i(t, S)$ to the N Hamilton-Jacobi-Bellman (HJB) equations related to the optimization problem (10), i.e.,

$$
\begin{aligned}
&\min_{r_i(t)}[C_i\left(t\right) - r_i\left(t\right)\partial_{Q_i} v_i^*\left(t, S_i(t)\right) + \alpha\left(t\right)\partial_{h_i} v_i^*\left(t, S_i(t)\right) \\
&+ \frac{1}{2}\sigma_b^2 \partial_{h_i h_i} v_i^*\left(t, S_i(t)\right)] + \partial_t v_i\left(t, S_i\left(t\right)\right) = 0.
\end{aligned} \tag{14}
$$

Proof: As $v_i\left(t, S_i\left(t\right)\right)$ is the value function of cost $C_i\left(t\right)$ at the state $S_i\left(t\right)$, according to Bellman's principle of optimality, increasing time t to $t + dt$, leads to

$$
\begin{aligned}
&v_i\left(t, S_i\left(t\right)\right) \\
&= \min_{r_i(t)} \mathbb{E}\left[\int_t^{t+dt} C_i\left(u\right)du + v_i\left(t + dt, S_i\left(t + dt\right)\right)\right].
\end{aligned} \tag{15}
$$

By adopting Taylor's expansion of $v_i\left(t + dt\right)$, we obtain

$$
\begin{aligned}
&v_i\left(t + dt, S_i\left(t + dt\right)\right) \\
&= v_i\left(t, S_i\left(t\right)\right) + [\partial_t v_i\left(t, S_i\left(t\right)\right) + \partial_t S_i\left(t\right) \cdot \nabla v_i\left(t, S_i\left(t\right)\right)]\,dt \\
&\quad + o\left(dt\right), .
\end{aligned} \tag{16}
$$

in which $\partial_t v_i$ is the first-order partial differential of v_i with respect to t, ∇v_i is the gradient of v_i with respect to the state variables $S_i(t)$, and $o\left(dt\right)$ denotes the terms of order higher than one in the Taylor expansion. By substituting (16) into (15), canceling $v_i\left(t, S_i\left(t\right)\right)$ on both sides, dividing both sides by dt, and taking the limit when dt approaches zero, we find that $o\left(dt\right)$ is negligible compared to other items, which leads to

$$
\begin{aligned}
&\min_{r_i(t)}\left[C_i\left(t\right) + \partial_t S_i\left(t\right) \cdot \nabla v_i\left(t, S_i\left(t\right)\right)\right] \\
&\qquad\qquad + \partial_t v_i\left(t, S_i\left(t\right)\right) = 0.
\end{aligned} \tag{17}
$$

By substituting (12) into (17), we obtain the HJB equation (14). ■

Theorem 1: There exists a Nash equilibrium for the multi-player non-cooperative dynamic stochastic game in (10).

Proof: The existence of a solution to the HJB equation ensures the existence of the Nash equilibrium for the game. It is known that there exists a solution to the HJB equation if the Hamiltonian is smooth [40]. Based on the HJB equation, the Hamiltonian is defined as

$$\begin{aligned} &H\left(r_i\left(t\right), S_i\left(t\right), \nabla v_i\left(t, S_i\left(t\right)\right)\right) \\ &\quad = \min_{r_i(t)} \left[C_i\left(t, S_i\left(t\right)\right) + \partial_t S_i\left(t\right) \cdot \nabla v_i\left(t, S_i\left(t\right)\right)\right]. \end{aligned} \quad (18)$$

The derivatives of all orders exist for the Hamiltonian due to the continuity of the defined cost function $C_i(t)$, $\alpha_i(t)$ and $\sigma_b(t)$, as well as the derivatives of the Hamiltonian with respect to $r_i(t)$. Due to the existence of the derivatives, the Hamiltonian is smooth. This concludes the proof. ■

Theorem 2: Since the optimal running cost trajectory $v_i^*(t, S_i(t))$ is the unique solution to the HJB equation (14), the optimal offloading strategy for SMD i can be obtained as:

$$r_i^*(t,S) = B\log_2\left[\frac{Bh_i\left(t\right)}{\eta\sigma_n^2 \ln 2}\left(\partial_{Q_i} v_i^*(t,S) - \Psi_i\left(t\right)\right)\right]. \quad (19)$$

Proof: Based on (14), the Hamiltonian can be rewritten as

$$\begin{aligned} &H\left(r_i\left(t\right), S_i\left(t\right), \nabla v_i\left(t, S_i(t)\right)\right) \\ &= \min_{r_i(t)} \left[\left(2^{\frac{r_i(t)}{B}} - 1\right)\frac{\eta\sigma_n^2}{h_i\left(t\right)} + \Psi_i(t) r_i(t)\right. \\ &\quad - r_i\left(t\right)\partial_{Q_i} v_i^*\left(t, S_i(t)\right) + \alpha\left(t\right)\partial_{h_i} v_i^*\left(t, S_i(t)\right) \\ &\quad \left. + \frac{1}{2}\sigma_b^2 \partial_{h_i h_i} v_i^*\left(t, S_i(t)\right)\right] \end{aligned} \quad (20)$$

Then, differentiating the infimum term in (20) with respect to $r_i(t)$ and equating it to zero lead to:

$$\frac{\eta\sigma_n^2 \ln 2}{h_i\left(t\right)B} 2^{\frac{r_i(t)}{B}} + \Psi_i(t) - \partial_{Q_i} v_i^*(t,S) = 0. \quad (21)$$

Therefore, by isolating $r_i(t)$ in (21), we can obtain the optimal $r_i^*(t,S)$ in (19). ■

However, the computation price $\Psi_i(t)$ in solution (18), as defined by (7), is decided by the collective decisions of all the SMDs, thus to find a Nash equilibrium for the N-user non-cooperative DSG requires to solve N coupled HJB equations (14) for each SMD. Even worse, to implement this approach in practice will need the N players to exchange extensive signaling messages, which becomes prohibitively complex when N grows large. To address this scaling challenge, we propose to transform the N-user non-cooperative DSG into a MFG which is more tractable.

IV. Mean Field Game Approach

As introduced in [41], [42], the MFGs can be seen as the extension of multi-user non-cooperative DSGs when the number of users is large enough to be considered infinite. In this section, we reformulate the optimization problem (10) with MFG, derive its mean field equilibrium with HJB and FPK equations, and detail the iterative algorithm employed to solve these PDEs, based on a finite difference method.

A. Mean Field Game Assumptions, Reformulations and Approximations

According to the mean field theory [20], a MFG model include one generic player, taking rational actions, and a mean field, representing the collective actions of all other players. When the game starts, the generic player will create a decision set for all possible states to optimize its performance target (or utility), and all the players will share this decision set since they are symmetrical in possible states and equally rational. Then the mean field will create the cumulative impact of all the rest players to the generic player, based on the probability density function (PDF) of their states and the shared decision set. The generic player will adjust its decision set according to the feedback of the mean field, and the mean field will create new impact according to the updated decision set again. This iterative process will continue until a NE is achieved. It is obvious that the converge time of MFG, as a 2-body game, will not increase with the number of players, thus the scaling problem of DSGs can be solved.

To formulate a MFG problem, the following four hypotheses (H1-H4) must be considered for the investigated computation offloading game:

- *H1- Rational Expectations and Behaviors of the Players:* The players' decisions are rational and based on the cost functions, which means that the players shall optimize their present offloading strategy according to both the current states, the prediction of future channel evolution, and the expected offloading decisions of all the other players, to minimize their cost functions.
- *H2 - Interchangeability of the States Among the Players:* Any permutation of two players does not change the global outcome of the game. In addition, two players sharing the identical state have exactly the same optimal offloading decision. Therefore, the index i of state $S_i(t)$ can be removed, and a unique MFG-based policy $r^*(t,S)$ can be defined. This policy applies to every player and only depends on the state $(t, S(t))$ that each player experiences. This greatly simplifies the game, since only one set of optimal offloading strategies need to be adopted by every player in any state, instead of N individual strategies.
- *H3 - The Existence of a Continuum of the Players:* As the number of players is large enough to be considered infinite, the large population of players can then be modeled as a continuum of players.
- *H4 - Social Interaction Between Players Through the Mean Field:* Each player interacts with the mean field instead of interacting with all the other players separately. From a single player's point of view, the interaction with the $N-1$ other players does not consist of one-to-one interactions, instead the player is affected by a joint response of the $N-1$ other players altogether, in the form of a dynamic computation price for each player.

Based on the hypotheses H1-H4, the mean field equivalent for the investigated optimization problem (10) can be defined in Definition 2 as below:

Definition 2: Given the state space $S(t) = [Q(t), h(t)]$ (H2), the mean field is a statistical distribution of this state space at time t, and the probability density of users in a specific state is defined as

$$m(t,S) = \lim_{N\to\infty} M(t,S) = \lim_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} \mathbb{1}_{S_i(t)=S}, \quad (22)$$

where $M(t,S)$ denotes the proportion of SMDs in state S at time $t \in \mathcal{T}$, and $\mathbb{1}$ is the indicator function which returns 1 if the given condition is satisfied, and zero otherwise. When the number of SMDs, N, goes to infinity, this proportion $M(t,S)$ converges to a mean field probability density function (PDF) $m(t,S)$ (as indicated in H3), which characterizes the state evolution of the SMDs over time. As $m(t,S)$ is a continuum PDF, it satisfies

$$\int_h \int_Q m(t,S) dh dQ = 1. \tag{23}$$

Next, we need to redefine the computation price term $\Psi(t)$ using the MFG-based offloading policy $r^*(t,S)$ and the mean field PDF $m(t,S)$, and prove that $\Psi(t)$ converges to a mean field price when the number of SMDs N goes to infinity.

The original price term $\Psi_i(t)$ is defined in (7) as a weighted sum of all the other SMDs' offloading strategies. Based on $M(t,S)$, the optimal offloading strategy $r^*(t,S)$, and $S(t) = (Q(t), h(t))$, (7) can be rewritten as

$$\Psi_i(t) = \lambda + \xi \left[\frac{N}{N-1} \int_h \int_Q M(t,S) r^*(t,S) dh dQ - \frac{1}{N-1} r_i(t,S) \right] \tag{24}$$

The mean field price term $\Psi(t)$ is then obtained from $\Psi_i(t)$, when N tends to infinity, i.e.,

$$\Psi(t) = \lim_{N\to\infty} \Psi_i(t) = \lim_{N\to\infty} \lambda + \xi \left[\frac{N}{N-1} \int_h \int_Q M(t,S) r^*(t,S) dh dQ - \frac{1}{N-1} r_i(t,S) \right] \tag{25}$$

In (25), when $N \to \infty$, $\frac{N}{N-1} \to 1$; $\lim_{N\to\infty} \frac{1}{N-1} r_i(t,S) = 0$ since $r_i(t,S)$ is bounded as given in (5). Then, the mean field price can be approximated by:

$$\Psi(t) \approx \lambda + \xi \left(\int_h \int_Q m(t,S) r^*(t,S) dh dQ \right). \tag{26}$$

B. Mean Field Game Optimization Problem

We have applied mean field approximation to transform the N-body problem in (10) into an equivalent MFG, which is a 2-body problem between a generic SMD and the mean field.

(1) *First Body - Generic SMD:* Assuming that all other players' strategies are fixed, one player in any state can compute its optimal offloading strategy from the HJB equation. It rewrites as

$$\min_{r(t)} [C(t,S) - r(t,S)\partial_Q v^*(t,S) + \alpha(t) \partial_h v^*(t,S) + \frac{1}{2}\sigma_b^2 \partial_{hh} v^*(t,S)] + \partial_t v^*(t,S) = 0 \tag{27}$$

in MFG based on (14). Then the best offloading decision $r^*(t,S)$ in (19) can be expressed as

$$r^*(t,S) = B \log_2 \left[\frac{Bh(t)}{\eta \sigma_n^2 \ln 2} \left(\partial_Q v^*(t,S) - \Psi(t) \right) \right]. \tag{28}$$

by using the mean field price $\Psi(t)$, in response to a given mean field $m(t,S)$. The optimal running cost trajectory $v^*(t,S)$ is solved with the HJB equation (27) by backward reasoning. Starting from the final boundary condition $v^*(T, S(T)) = F(Q(T))$ to derive $v^*(T-1,S), v^*(T-2,S), \ldots, v^*(0,S)$, respectively.

(2) *Second Body - Mean Field:* The evolution of the mean field $m(t,S)$ can be described using the FPK equation [20], when all users in the system implement the mean field strategy $r^*(t,S)$. Given the offloading strategy $r^*(t,S)$, the evolution of the mean field density $m(t,S)$ can be obtained by solving the FPK equation (29) in forward direction, starting from the initial boundary condition $m(0,S)$.

$$\partial_t m(t,S) + \partial_h (\alpha(t) m(t,S)) - \partial_Q (r(t,S) m(t,S)) - \frac{1}{2}\sigma_b^2 \partial_{hh} m(t,S) = 0 \tag{29}$$

Proof: Let $y(S)$ be a smooth and compactly supported function, then the integral $\int m(t,S) y(S) dS$ can be seen as the continuum limit of the sum $\frac{1}{N}\sum_{i=1}^{N} y(S_i)$, i.e.,

$$\int m(t,S) y(S) dS \approx \frac{1}{N} \sum_{i=1}^{N} y(S_i(t)). \tag{30}$$

By differentiating both sides of (30) with respect to t and applying the chain rule, we have

$$\partial_t m(t,S) y(S) dS \approx \frac{1}{N} \sum_{i=1}^{N} \left[\partial_t S_i(t) \nabla y(S_i) + \partial_t^2 S_i(t) \Delta y(S_i) \right], \tag{31}$$

where $\nabla y(S_i)$ and $\Delta y(S_i)$ are the gradient and Laplacian of the function y along S_i, respectively. When $N \to \infty$, (31) converts to

$$\partial_t m(t,S) y(S) dS = \int \left[\partial_t S(t) \nabla y(S) + \partial_t^2 S(t) \Delta y(S) \right] m(t,S) dS. \tag{32}$$

Applying integration by parts on (32) leads to

$$\left[\partial_t m(t,S) + \partial_t S \nabla m(t,S) - \partial_t^2 S \Delta m(t,S) \right] y(S) dS = 0, \tag{33}$$

which is valid for any smooth function. Letting $y(S) = 1$, we have

$$\partial_t m(t,S) + \partial_t S(t) \nabla m(t,S) - \partial_t^2 S(t) \Delta m(t,S) = 0 \tag{34}$$

By substituting the state $S(t) = [Q(t), h(t)]$ into (34), the FPK equation (29) is obtained. ■

To this end, we have transformed the N-body DSG into a 2-body MFG. The two bodies here consist of our generic SMD, and a mean field continuum including the large mass of SMDs competing against the generic SMD. Fig. 2 provides a graphical representation of the two body interaction, in which the core idea is that the HJB equation allows to compute the optimal offloading strategies to be used for any SMD in any states (channel, remaining task data, etc), in response to a given mean field price, whereas the FPK allows to define the mean field price response, if all SMDs in the system implement the MFG offloading strategy.

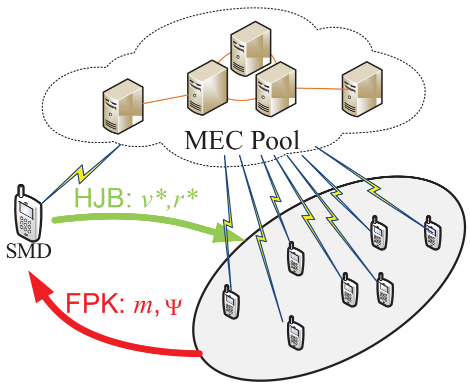


Fig. 2. A graphical explanation of how the two body MFG works for N-body computation offloading decisions.

Discussion:

1) Efficiency of MFG approach: Mean field equilibrium (MFE) can be seen as equivalent to the Nash equilibrium for DSGs. For MFGs, the MFE represents the stable combination of both the value function $v(t,S)$ and the mean field $m(t,S)$. At any time t and state S, the value function $v(t,S)$ and the mean field $m(t,S)$ interact with each other, where the optimal running cost trajectory $v^*(t,S)$ is the solution to the HJB equation in (27) and $m(t,S)$ is the solution of the FPK equation (29). $v^*(t,S)$ determines $r^*(t,S)$ in (28), then affects the evolution of the mean field via (29); $m(t,S)$ determines the mean field price ψ in (26), which affects $v^*(t,S)$ through (27). Therefore, the optimal offloading strategy can be obtained by solving the two coupled forward-backward PDEs iteratively. It is guaranteed that the iterative algorithm will converge and lead to the optimal mean field strategies, as all the involved functions are smooth [44]. Also, it will converge fast [45], as the iterative algorithm includes only two bodies, instead of N bodies if without the mean field game.
2) How large should N be to adopt MFG: It is assumed that the number of SMDs is very large to meet H3 (The existence of a continuum of the players) to apply the mean field theory. However, it has been verified by the simulations in [15] that even when N is as small as 100, the mean field model is still accurate enough to approximate the interactions among a group of players, which is easy to be satisfied in the UDN scenario.

C. Proposed Algorithm With Finite Difference Method

There is no closed-form expressions for the exact solutions of (19) and (29), thus we adopt finite difference methods (FDM) [43], [46] to numerically solve the coupled HJB and FPK equations. With FDM, the investigated offloading time interval $[0,T]$, the remaining task data space $[0,D]$ and the channel state $[h_{min}, h_{max}]$ are discretized with the steps of $\Delta t, \Delta Q$ and Δh, respectively. Let n, q, l be the index of discrete time, remaining data size, and channel state, thus $t = n\Delta t, Q = q\Delta Q$ and $h = l\Delta h$. In this setting, the finite difference expressions of the first and second order derivatives of the continuous MFG with respect to time and state space can be written as:

Algorithm 1 The Iterative Method for Computing the MFE Rate Offloading Strategy

Initialize: $m_{(0)}$, $v^*_{(0)}$, $\mathbf{r}^*_{(\mathbf{0})}$, $\Psi_{(0)}$, $k=0$, k_{max}, ε.
while $k < k_{max}$ **do**
  $k = k+1$
  Using $\mathbf{r}^*_{(k-1)}$, solve the HJB equation for $v^*_{(k)}$ with (39).
  Using $v^*_{(k)}$ and $\Psi_{(k-1)}$, update $\mathbf{r}^*_{(k)}$ with (28).
  if $\left|\mathbf{r}^*_{(k)}(n,q,l) - \mathbf{r}^*_{(k-1)}(n,q,l)\right| < \varepsilon$ **then**
    break;
  end if
  Using $\mathbf{r}^*_{(k)}$, solve the FPK equation for $m^*_{(k)}$ with (40).
  Using $m_{(k)}$ and $\mathbf{r}^*_{(k)}$, update $\Psi_{(k)}$ with (26).
end while

$$\frac{\partial v^*(t,Q,h)}{\partial t} \approx \frac{v^*(n+1,q,l) - v^*(n,q,l)}{\Delta t}, \tag{35}$$

$$\frac{\partial v^*(t,Q,h)}{\partial q} \approx \frac{v^*(n,q,l) - v^*(n,q-1,l)}{\Delta q}, \tag{36}$$

$$\frac{\partial v^*(t,Q,h)}{\partial h} \approx \frac{v^*(n,q,l+1) - v^*(n,q,l)}{\Delta h}, \tag{37}$$

$$\frac{\partial^2 v^*(t,Q,h)}{\partial h^2} \approx \frac{v^*(n,q,l+1) - 2v^*(n,q,l) + v^*(n,q,l-1)}{\Delta h^2}. \tag{38}$$

By substituting (35) - (38) into (27), the HJB equation can be rewritten in a backward discrete form as

$$\begin{aligned} v^*(n,q,l) = & \left[1 + \Delta t\left(\frac{r(n)}{\Delta q} + \frac{\alpha(n)}{\Delta h} + \frac{\sigma_b^2}{\Delta h^2}\right)\right]^{-1} \\ & \times \left[v^*(n+1,q,l) + \left(\frac{r(n)\Delta t}{\Delta q}\right) v^*(n,q-1,l)\right. \\ & + \frac{\sigma_b^2 \Delta t}{2\Delta h^2} v^*(n,q,l-1) + \Delta t\left(\frac{\alpha(n)}{\Delta h} + \frac{\sigma_b^2}{2\Delta h^2}\right) \\ & \left.\times v^*(n,q,l+1) + \Delta t C(n)\right]. \end{aligned} \tag{39}$$

When $n = T, v^*(n+1,q,l) = F(Q(T))$. Similarly, the FPK equation can be rewritten as

$$\begin{aligned} mm(n+1,q,l) = & \frac{1}{2}[m(n,q+1,l) \\ & + m(n,q-1,l) + m(n,q,l+1) \\ & + m(n,q,l-1)] - \frac{\Delta t}{2h}[\alpha(n,q,l+1) \\ & \times m(n,q,l+1) - \alpha(n,q,l-1) \\ & \times m(n,q,l-1)] + \frac{\Delta t}{2\Delta Q}[r(n,q+1,l) \\ & \times m(n,q+1,l) - r(n,q-1,l) \\ & \times m(n,q-1,l)] + \frac{1}{2}\sigma_b^2 \frac{\Delta t}{(\Delta h)^2} \\ & \times [m(n,q,l+1) - 2m(n,q,l) \\ & + m(n,q,l-1)]. \end{aligned} \tag{40}$$

To this end, the iterative method to obtain the MFE rate offloading strategy (MFEROS) is described in **Algorithm 1**.

In **Algorithm 1**, $Y_{(k)}$ denotes the value of $Y = \{m, v, \mathbf{r}^*, \Psi\}$ at iteration k; ε is the convergence threshold, which is a relatively small value; k_{max} is the maximum allowed number of iterations. At the beginning of the algorithm, $v^*_{(0)}$, $\mathbf{r}^*_{(0)}$ and $\Psi_{(0)}$ are initialized as zeros, and the evolution density $m_{(0)}$ is given as a specific known distribution. Then, the algorithm solves (39) and (40) iteratively and updates the parameters in Y, until $\mathbf{r}^*$ converges or k_{max} is reached. Since the initial condition of the system is given in Algorithm 1, then the proposed MFEROS scheme will be able to obtain the optimal decision set for any SMD in any state (i.e., time, channel condition, remaining data size). Also, thanks to the MFG framework, Algorithm 1 can be computed by any SMD in a distributed way.

Besides, a penalty function $F(Q(T))$ must be defined for the MFEROS, which should return zero when $Q(T) = 0$, and a large value when $Q(T) > 0$. We select a parametric logistic function as penalty, i.e.,

$$F\left(Q\left(T\right)\right) = \frac{\varphi}{1 + e^{-\rho Q(T)}} - \frac{\varphi}{2}. \tag{41}$$

V. Numerical Results

In this section, the performance of the proposed MFE rate offloading strategy are numerically evaluated in comparison with some other offloading strategies as follows:

- *S1 - MFE Rate Offloading Strategy (MFEROS):* the offloading strategy is obtained by computing the mean field equilibrium in Algorithm 1.
- *S2 - Maximal Rate Offloading Strategy (MROS):* the SMDs offload tasks with the maximum transmit rate (or offload tasks at the maximal power), until the offloading is complete.
- *S3 - Constant Rate Offloading Strategy (CROS):* the SMDs offload at a constant data rate to fully utilize the whole available time duration, no matter what the channel gain and computation price is [16].
- *S4 - Time-Domain Water-Filling Offloading Strategy (TWFOS):* S4 has the same cost function as S1, but it is given perfect knowledge about the channels' evolution during the whole offloading duration, thus time-domain water-filling [21] is adopted to decide the optimal instantaneous offloading rate, while the dynamic computation price for each SMD is solved via a traditional iterative way. Therefore, this strategy has huge computational complexity, making it impossible to be adopted in practice when the number of SMDs grows large. Nevertheless, we present it as a benchmark to show the optimal performance bound under perfect future channel information.

In practical wireless communication systems, transmissions are fulfilled in discrete time slots. Therefore, following the optimization target in (10), we define the performance metric in simulation as the *cumulated cost* averaged among all SMDs in discrete form as :

$$\Omega = \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} C_i\left(t\right), \tag{42}$$

in which $\sum_{t=1}^{T} C_i\left(t\right)$ is the discrete form of the target cost function in (10).

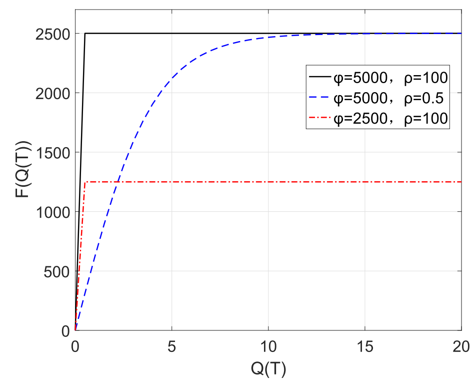


Fig. 3. The penalty function versus $Q(T)$ for 3 different pairs of φ and ρ.

TABLE I
SIMULATION PARAMETERS

Number of time slots	20
Time slot duration Δt	$0.5ms$
Initial data size D	$2.5 \times 10^5 bits$
Data grid resolution ΔQ	$5 \times 10^3 bits$
Channel grid resolution Δh	3×10^{-4}
Channel bandwidth B	$5MHz$
Maximal power p^{max}	$23dBm$
Noise power σ_n^2	$-104dBm$
Basic price λ	10^{-8}

Fig. 3 illustrates the behavior of the penalty function defined in Eq. (41) under different parameters settings, where parameter φ and ρ respectively define the maximum value and the steepness of the curve. We observe that parameter φ defines the maximal penalty value the function takes, whereas parameter ρ defines the steepness of the curve.

In the following simulations, $\varphi = 5000$, $\rho = 100$ and other major simulation parameters are listed in Table I.

A. Semi-Static Channels

In this subsection, we consider a particular case where the channels are semi-static during the whole offloading duration, i.e., α and σ_b in (1) are set to zero. As a consequence, the state $S(t)$ consists only of the remaining packet size to be transmitted, $Q(t)$.

Fig. 4 shows the evolution of the optimal offloading strategy $r^*(t, Q)$ for each SMD with respect to time and the remaining data size. When the time state is fixed, it can be observed that the value of $r^*(t, Q)$ increases when the remaining data Q becomes large. However, the value of $r^*(t, Q)$ also increases over time when the data state is fixed. The reason is, when larger remaining data volume need to be offloaded, or fewer time slots are left for transmission, therefore forcing the system to transmit at higher rates during the remaining time slots, as to complete offloading process before the deadline.

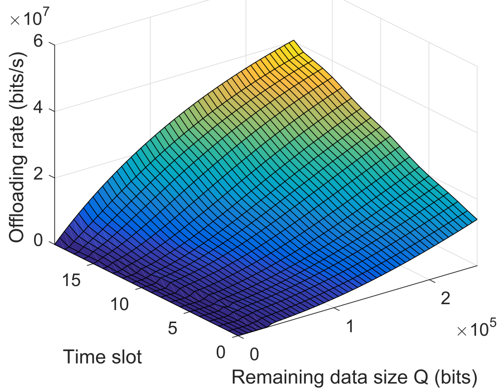


Fig. 4. The optimal instantaneous offloading rate $r^*(t, Q)$ for an SMD versus the state (t, Q).

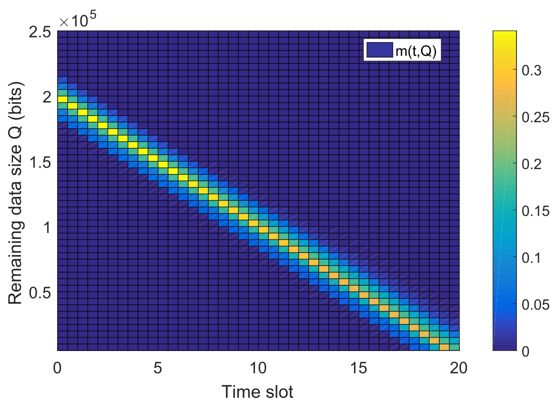


Fig. 5. Evolution of the mean field $m(t, Q)$ distribution.

Fig. 5 illustrates the evolution of the mean field distribution $m(t, Q)$ when the channel gain is constant. Note that $m(t, Q)$ indicates the density of SMDs in state (t, Q). The initial distribution of the mean field $m(0, Q)$ follows a normal distribution $\mathcal{N}(8 \times 10^6, 2.5 \times 10^{11})$. In this data size distribution, the expected remaining data volume decreases with time which corresponds to the trajectory of the optimal offloading rate in Fig. 4. Since the channel gain is constant, it can be observed that the trajectory of the peak of density $m(t, Q)$ is almost a straight line. This follows from the optimal MFEROS strategy being constant over time, as the channels remain constant over time as well, as shown in Fig. 6 and Fig. 7. At the deadline, almost every SMD has completed its offloading as requested, while a little proportion of them have very small data volume left, and accept the penalty F(Q(T)) for this small amount of remaining data to be transmitted. This phenomenon is unfortunately unavoidable, but its effect is negligible and thus considered acceptable.

In Fig. 6 and Fig. 7, we present the cumulated cost and evolution of remaining data size for a generic SMD for offloading strategies S1, S2 and S3, respectively. Under static channel gain, strategy S4 employs a constant offloading rate (which can be proven easily with KKT conditions), thus it is equivalent to strategy S3. This is the reason why we do not present the results of strategy S4 in this subsection.

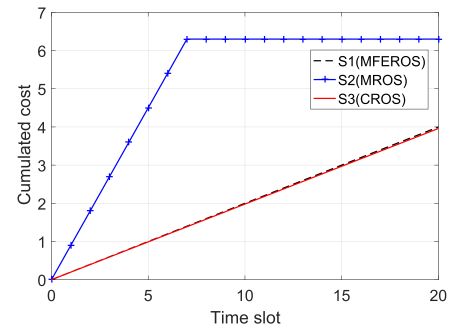


Fig. 6. The cumulated cost of an SMD versus time slot (t) under 3 offloading strategies.

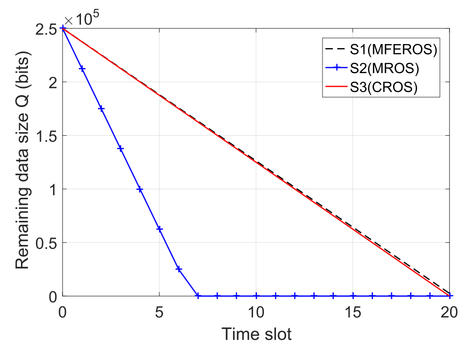


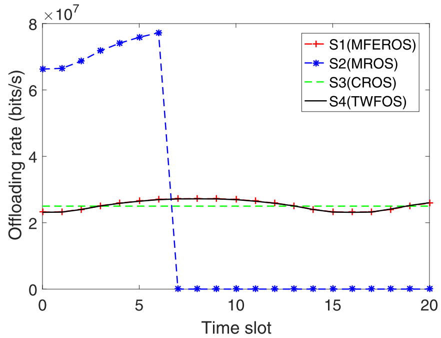
Fig. 7. The remaining data size $Q(t)$ of an SMD versus time slot (t) under 3 offloading strategies.

In Fig. 6, it can be observed that the cumulated cost of strategy S1 and strategy S3 are closely equivalent. This demonstrates that strategy S1 can be used to closely approximate the optimal strategy S3. Due to the full power transmitting and the large computation price, compared with strategy S1, the strategy S2 has highest cumulated cost.

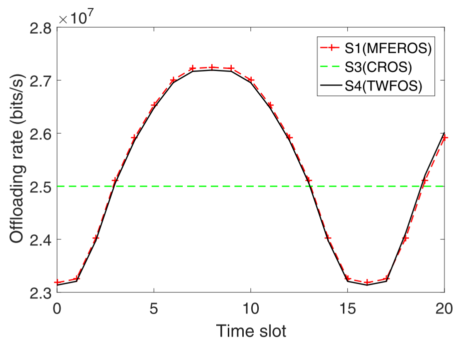
In Fig. 7, it appears that the remaining data size evolution of strategy S1 and strategy S3 are almost overlap with each other. As strategy S3 is known to be the optimal offloading strategy, it means that the proposed strategy S1 is able to approach notably the optimal offloading strategy. At the same time, we can observe that the strategy S1 offloads the same amount of data on every single time slot, and so dose the optimal offloading strategy S3. On the contrary, in order to avoid the penalty at the last time slot, the strategy S2 offloads the data as soon as possible. While the applications investigated in this work require to be completed before a given deadline with the minimum possible energy consumption and computation cost, S2 allows for an early completion of the data offloading as revealed in Fig. 7, which does not fully exploit the available time duration for offloading and results in the highest cumulated cost as shown in Fig. 6.

B. Time-Varying Channels

In this subsection, we investigate the computation offloading performance under time-varying channels with no uncertainty, i.e. $\sigma_b = 0$, meaning that the channels are fully predictable. For demonstration purposes, the channel variation is



(a) Offloading Rate VS Time



(b) Offloading Rate VS Time (zoomed-in version)

Fig. 8. The instantaneous offloading rate for a generic SMD versus time slot (t) under different offloading strategies.

modeled as

$$h\left(t\right) = h\left(0\right) + A\sin\left(f_0 t\right), \tag{43}$$

where $h(0) = 2 \times 10^{-3}$, $f_0 = 0.4$ and $A = 10^{-3}$. In this particular case, $\alpha(t)$ in (1) can be described as

$$\alpha\left(t\right) = Af_0\cos\left(f_0 t\right). \tag{44}$$

Fig. 8 illustrates the instantaneous offloading rate decisions for a generic SMD, under different strategies. As shown in Fig. 8(a), the full power strategy S2 completes the transmission notably in advance, using a high offloading rate. Once completed, it simply stops offloading and becomes dormant. Therefore, it is unable to exploit the latency constraint efficiently, which is given in (3), nor is it able to exploit any available knowledge about future channels. For readability, we present a zoomed-in version of Fig. 8(a) in Fig. 8(b), with the strategies S1, S3 and S4 only. In Fig. 8(b), although S3 can utilize all the time slots, due to its constant offloading rate, it can not adapt to the varied channel states and the dynamic computation price evolution. Contrarily, the proposed MFE offloading strategy S1 can not only exploit the latency constraint, but also adapt to the channel variations, therefore closely approaching the optimal performance of strategy S4.

Fig. 9 presents how the cumulated cost evolves with time. At the last time slot, it can be observed that the proposed strategy S1 has minimal cumulated cost, which is overlapping with the optimal strategy S4. This means that S1 nicely approximates the optimal strategy S4. Due to the full power transmission policy and the high computation price at the MEC

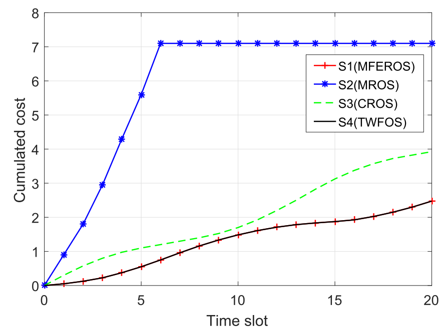


Fig. 9. The cumulated cost versus time slot (t) under 4 offloading strategies.

pool, the strategy S2 has maximal cumulated cost. For S3, each SMD's transmission rate is evenly allocated to all time slots, thus their transmission power cost and computation cost are constant at each time slot (since the computation load at MEC is constant, the computation price is also constant according to (7)), and the resulted cumulated cost is much lower than S2 but still higher than S1/S4. The reason that S1 can significantly reduce the cumulated cost is as follow. In the considered dynamic computation offloading scenario, the combined SMD transmission energy cost and MEC computation cost become high when the wireless channel condition is poor and/or many SMDs are offloading simultaneously. In order to minimize the cost function while meeting the given deadline, the proposed S1 allows an SMD to transmit more of its task bits to the MEC when its channel condition is better and/or the computation price is lower, and transmit less when its channel condition is worse and/or the computation price is higher, based on its prediction of the channel variation and the offloading decisions of other SMDs.

From Fig. 8 and Fig. 9, we can summarize that the performance gains of S1 over S2 and S3 are twofold:

First, S1 Offers a "Latency Gain" (as Defined in [21]) Over S2: This gain follows from the fact that S1 allows an SMD to exploit the time periods when the channel condition is good and/or few other SMDs are offloading during the entire time within the latency constraint for computation offloading, whereas S2 attempts to complete the transmission as soon as possible without considering the varying channel condition or other SMDs' offloading decisions, which increases the transmission cost and computational cost, since the computational price increases for a high instantaneous offloading rate.

Second, S1 Offers a "Future Channel Knowledge Gain" Over S3: S1 enables a SMD to adapt the offloading rate within every time slot to the present and the predicted future link quality, whereas S3 adopts a constant transmission rate for offloading. Thus, S1 achieves a gain over S3 by exploiting the predicted knowledge about the future transmission context.

C. Stochastic Channels

As introduced in (1), the random part of channels is modeled as independent Wiener processes with variance σ_b^2. In this subsection, we investigate how the channel uncertainty parameter σ_b affects the performance of the proposed strategy S1 by considering the following scenarios:

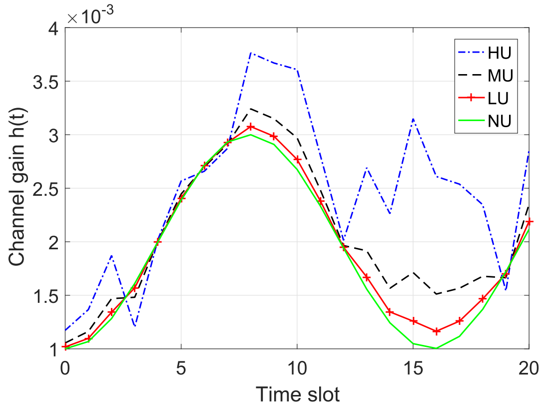
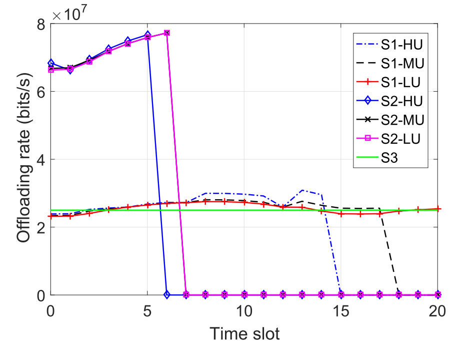
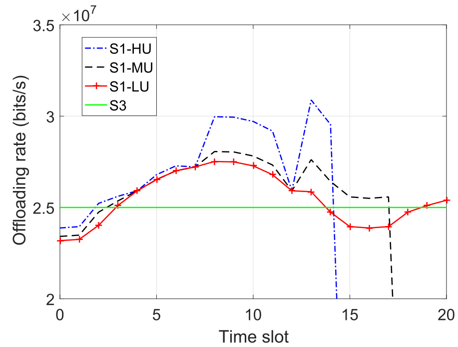


Fig. 10. Channel evolution with different levels of stochasticity.



(a) Offloading Rate VS Time



(b) Offloading Rate VS Time (zoomed-in version)

Fig. 11. Instantaneous offloading rate under different channel uncertainty scenarios.

- *Low Uncertainty (LU):* $\sigma_b = 0.1$;
- *Mid Uncertainty (MU):* $\sigma_b = 1$;
- *High Uncertainty (HU):* $\sigma_b = 10$;

If we use $\sigma_b \Delta t / h_0$ to measure the channel randomness, then LU, MU and HU correspond to $\sigma_b \Delta t / h_0 = 2.5\%, 25\%$ and 250%, respectively. Fig. 10 presents the channel evolution with different levels of stochasticity, in which the green curve stands for No uncertainty (NU): $\sigma_b = 0$, where the system has perfect knowledge about the channel evolution. As expected, the higher the uncertainty parameter σ_b, the more the channel realization deviates from the exact trajectory (no uncertainty scenario), and the more unpredictable the future link quality becomes. In particular, for high values of σ_b, the prediction can become very uncertain and the trajectory for the future channels can become highly unpredictable.

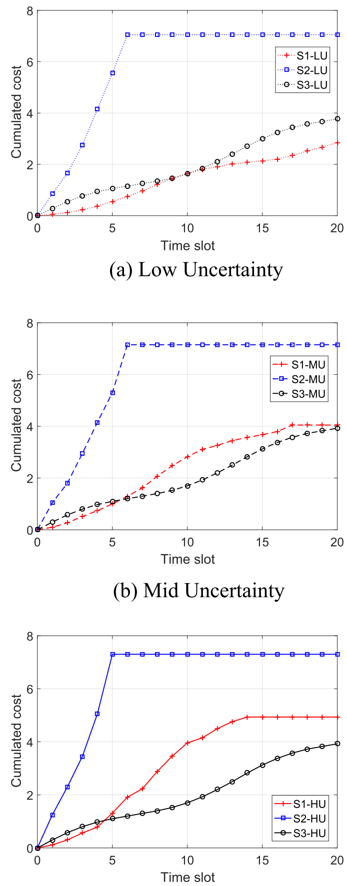
We compare the instantaneous offloading rate for a generic SMD with S1, S2 and S3 in Fig. 11, to reveal the impact of channel uncertainty (Since S4 has the perfect knowledge about the channel evolution, its instantaneous offloading behavior should be highly similar to the NU scenario). For readability, a zoomed-in version of Fig. 11(a) is presented in Fig. 11(b). In the numerical results, S3 is not be affected by the channel randomness at all, so we use only one curve to represent it. The performance curves of S1 and S2 in are all labeled with HU, MU and LU for the different channel randomness scenarios. No matter what channel scenario is, S2 always completes the computation offloading notably in advance, similar to the Time-Varying Channels without randomness. It is interesting to notice that in LU, S1's behavior is similar to that in NU scenario (Fig.8), but as the channel uncertainty increase to MU and HU, S1 increase its offloading rate and finish before the deadline T; the higher channel uncertainty (HU V.S. MU) is, the higher offloading rate is employed by S1, resulting in earlier completeness of the offloading process. The reason for that behavior is that in the highly uncertain channels, S1 will select higher offloading rate than actually needed to avoid the final penalty, since it is not sure about whether it can finish before the deadline.

Fig. 12 illustrates the cumulated cost for a generic SMD under different uncertainty scenarios. It can be observed that in Fig. 12, S1 achieves the lowest total cost when the offloading finishes in LU channels, almost equal total cost as S3 in MU channels, but higher cost than S3 in HU case. This is because S1 depends on the accurate predication of future channels to optimize its offloading decisions, as the channel uncertainty rises, it prefers to offload as soon possible to avoid having to transmit in an uncertain future, which significantly increases its cost for both transmission and computation. S2 always leads to highest cost in all three scenarios. Therefore, S1 should be applied when the channel prediction is relatively accurate, while the high uncertainty of the future channels will dramatically degrade the performance of S1.

D. Design Insights of the MFEROS

Since the proposed scheme works for an infinite number of players, its processing delay or required caching space of it does not scale with the input data size or the number of players. However, on the implementation side, since the MFE-based optimum has to be calculated using the Finite Difference Method (FDM), the processing delay of the proposed scheme depends on the selected resolution of the elements in FDM, e.g., the values of $\Delta t, \Delta Q, \Delta h$. If their values are too big, then the algorithm may not converge; while if they are too fine-grained, then the processing delay will be too large.

On the other hand, one may argue that in Figs. 7 and 8, S2 finishes the job faster than the proposed algorithm therefore the next set of jobs can be done right away, or the required deadline might be extend to gain even lower cost. For the first concern, this work focus on the applications that need



(a) Low Uncertainty

(b) Mid Uncertainty

(c) High Uncertainty

Fig. 12. The cumulated cost under different channel uncertainty scenarios.

to be completed before a given deadline with the minimum possible energy consumption and computation cost, similar to the applications investigated in [50]–[52]. For the second concern,

Accordingly, we formulate the optimization problem to minimize a cost function that combines the energy consumption of the smart mobile device (SMD) and the computation price (to pay to the MEC) subject to a fixed deadline, where the deadline cannot be extended as it is determined by the required Quality of Service and the objective is not to minimize the task completion time or maximize the transmission rate. In our considered dynamic computation offloading scenario, the combined transmission energy cost and computation cost at the MEC becomes high when the wireless channel condition is poor and/or when many SMDs are offloading simultaneously. In order to minimize the cost function while meeting the given deadline, the proposed MFG algorithm allows an SMD to transmit more of its task bits to the MEC when its channel condition is better and/or the computation price is lower, and transmit less when its channel condition is worse and/or the computation price is higher, based on its prediction of the channel variation and the offloading decisions of other SMDs.

VI. Conclusion and Future Work

In this paper, we have proposed a close-to-optimal computation offloading policy for the SMDs in an ultra-dense network while taking into account of both the task data state and the dynamic channel state. First, a multi-player non-cooperative DSG is defined to obtain the minimum cost for each SMD. In order to simplify the computation complexity of the Nash equilibrium for the game, a more tractable approach based on the MFG theoretic framework is proposed. Based on the mean field approximation, the coupled HJB and FPK partial differential equations are derived. Then, a numerical finite difference method is proposed to achieve the MFE in an iterative manner. Finally, numerical results reveal that the proposed MFE offloading strategy yields a much lower cumulated cost than the existing strategies under the given latency constraint. Our results also show that the performance of the MFE rate offloading strategy is sensitive to the uncertainty in the prediction of future channel states.

In our future work, we will consider the following possible leads:

Interference Management: in this work, uplink intra-cell or inter-cell interference is not included for simplicity. Actually, the interference management in the ultra-dense network is extremely important, therefore the impact of interferences shall be included in the future work.

General Channel Model: in the problem formulation, we use a simplified hypothesis for the channel variation model, but our proposed method would still apply if $\alpha_i(t)$ and $\sigma_i(t)$ were different among SMDs, which scenario shall be detailed in our future work.

General Traffic Model: in this work, the computation task is assumed to be static, and the computation offloading periods are synchronized. However, in practical networks, the computation tasks at SMDs may be dynamic, and the traffic could be fluctuating both spatially and temporally, which may affect the proposed algorithm. In our future work, we will investigate more realistic scenarios where the computation tasks arrive continuously and the computation offloading periods are unsynchronized and will study the effect of unsaturated traffic [47] and [48].

Limited MEC Computation Capacity: In the current computation model, we assumed that the MEC pool has sufficient processing power, which means the latency for MEC computation and queueing can be neglected. In our future work, the MEC pool with limited computation capacity will be considered, and the MEC processing delay and queueing delay will be included in the optimization problem.

References

[1] R. Zheng, H. Wang, and M. De Mari, "Optimal computation offloading with a shared MEC center: A mean field game approach," in *Proc. IEEE Globecom Workshops (GC Wkshps)*, Waikoloa, HI, USA, Dec. 2019, pp. 1–6.

[2] S. Wang, X. Zhang, Y. Zhang, L. Wang, J. Yang, and W. Wang, "A survey on mobile edge networks: Convergence of computing, caching and communications," *IEEE Access*, vol. 5, pp. 6757–6779, Mar. 2017.

[3] J. Zhang, H. Guo, and J. Liu, "Energy-aware task offloading for ultra-dense edge computing," in *Proc. IEEE Int. Conf. Internet Things (iThings) IEEE Green Comput. Commun. (GreenCom) IEEE Cyber, Phys. Social Comput. (CPSCom) IEEE Smart Data (SmartData)*, Halifax, NS, Canada, Jul. 2018, pp. 720–727.

[4] M. Chen and Y. Hao, "Task offloading for mobile edge computing in software defined ultra-dense network," *IEEE J. Sel. Areas Commun.*, vol. 36, no. 3, pp. 587–597, Mar. 2018.

[5] N. Abbas, Y. Zhang, A. Taherkordi, and T. Skeie, "Mobile edge computing: A survey," *IEEE Internet Things J.*, vol. 5, no. 1, pp. 450–465, Feb. 2018.

[6] X. Shan, H. Zhi, P. Li, and Z. Han, "A survey on computation offloading for mobile edge computing information," in *Proc. IEEE 4th Int. Conf. Big Data Secur. Cloud (BigDataSecurity)*, Omaha, NE, USA, May 2018, pp. 248–251.

[7] F. Wang, J. Xu, and Z. Ding, "Multi-antenna NOMA for computation offloading in multiuser mobile edge computing systems," *IEEE Trans. Commun.*, vol. 67, no. 3, pp. 2450–2463, Mar. 2019.

[8] Y. Dai, D. Xu, S. Maharjan, and Y. Zhang, "Joint computation offloading and user association in multi-task mobile edge computing," *IEEE Trans. Veh. Technol.*, vol. 67, no. 12, pp. 12313–12325, Dec. 2018.

[9] H. Guo, J. Zhang, J. Liu, and H. Zhang, "Energy-aware computation offloading and transmit power allocation in ultradense IoT networks," *IEEE Internet Things J.*, vol. 6, no. 3, pp. 4317–4329, Jun. 2019.

[10] X. Chen, L. Jiao, W. Li, and X. Fu, "Efficient multi-user computation offloading for mobile-edge cloud computing," *IEEE/ACM Trans. Netw.*, vol. 24, no. 5, pp. 2795–2808, Oct. 2016.

[11] M. Liu and Y. Liu, "Price-based distributed offloading for mobile-edge computing with computation capacity constraints," *IEEE Wireless Commun. Lett.*, vol. 7, no. 3, pp. 420–423, Jun. 2018.

[12] H. Cao and J. Cai, "Distributed multiuser computation offloading for cloudlet-based mobile cloud computing: A game-theoretic machine learning approach," *IEEE Trans. Veh. Technol.*, vol. 67, no. 1, pp. 752–764, Jan. 2018.

[13] J. Zheng, Y. Cai, Y. Wu, and X. Shen, "Dynamic computation offloading for mobile cloud computing: A stochastic game-theoretic approach," *IEEE Trans. Mobile Comput.*, vol. 18, no. 4, pp. 771–786, Apr. 2019.

[14] C. Phillips, D. Sicker, and D. Grunwald, "A survey of wireless path loss prediction and coverage mapping methods," *IEEE Commun. Surveys Tuts.*, vol. 15, no. 1, pp. 255–270, 1st Quart., 2013.

[15] S. Samarakoon, M. Bennis, W. Saad, M. Debbah, and M. Latva-Aho, "Ultra dense small cell networks: Turning density into energy efficiency," *IEEE J. Sel. Areas Commun.*, vol. 34, no. 5, pp. 1267–1280, May 2016.

[16] M. De Mari, R. Couillet, E. C. Strinati, and M. Debbah, "Concurrent data transmissions in green wireless networks: When best send one's packets?" in *Proc. Int. Symp. Wireless Commun. Syst. (ISWCS)*, Paris, France, Aug. 2012, pp. 596–600.

[17] R. Bellman, "Dynamic programming and stochastic control processes," *Inf. Control*, vol. 1, no. 3, pp. 228–239, Sep. 1958.

[18] N. L. Stokey, *The Economics of Inaction: Stochastic Control Models With Fixed Costs*. Princeton, NJ, USA: Princeton Univ. Press, 2009.

[19] T. Basar and G. J. Olsder, *Dynamic Noncooperative Game Theory*, vol. 200. Philadelphia, PA, USA: SIAM, 1995.

[20] J.-M. Lasry and P.-L. Lions, "Mean field games," *Jpn. J. Math.*, vol. 2, no. 1, pp. 229–260, 2007.

[21] M. de Mari, E. C. Strinati, M. Debbah, and T. Q. S. Quek, "Joint stochastic geometry and mean field game optimization for energy-efficient proactive scheduling in ultra dense networks," *IEEE Trans. Cognit. Commun. Netw.*, vol. 3, no. 4, pp. 766–781, Dec. 2017.

[22] C. Yang, H. Dai, J. Li, Y. Zhang, and Z. Han, "Distributed interference-aware power control in ultra-dense small cell networks: A robust mean field game," *IEEE Access*, vol. 6, pp. 12608–12619, 2018.

[23] Y. Zhang, C. Yang, J. Li, and Z. Han, "Distributed interference-aware traffic offloading and power control in ultra-dense networks: Mean field game with dominating player," *IEEE Trans. Veh. Technol.*, vol. 68, no. 9, pp. 8814–8826, Sep. 2019.

[24] C. Yang, J. Li, P. Semasinghe, E. Hossain, S. M. Perlaza, and Z. Han, "Distributed interference and energy-aware power control for ultra-dense D2D networks: A mean field game," *IEEE Trans. Wireless Commun.*, vol. 16, no. 2, pp. 1205–1217, Feb. 2017.

[25] Y. Wang, F. R. Yu, H. Tang, and M. Huang, "A mean field game theoretic approach for security enhancements in mobile ad hoc networks," *IEEE Trans. Wireless Commun.*, vol. 13, no. 3, pp. 1616–1627, Mar. 2014.

[26] R. A. Banez, L. Li, C. Yang, L. Song, and Z. Han, "A mean-field-type game approach to computation offloading in mobile edge computing networks," in *Proc. IEEE Int. Conf. Commun. (ICC)*, Shanghai, China, May 2019, pp. 1–6.

[27] C. Bertucci, S. Vassilaras, J.-M. Lasry, G. S. Paschos, M. Debbah, and P.-L. Lions, "Transmit strategies for massive machine-type communications based on mean field games," in *Proc. 15th Int. Symp. Wireless Commun. Syst. (ISWCS)*, Lisbon, Portugal, Aug. 2018, pp. 1–5.

[28] M. Larranaga, M. Assaad, and K. DeTurck, "Queue-aware energy efficient control for dense wireless networks," in *Proc. IEEE Int. Symp. Inf. Theory (ISIT)*, Vail, CO, USA, Jun. 2018, pp. 1570–1574.

[29] P. Yen, Q. Zhan, and H. Minn, "New fractional frequency reuse patterns for multi-cell systems in time-varying channels," *IEEE Wireless Commun. Lett.*, vol. 4, no. 3, pp. 253–256, Jun. 2015.

[30] Z. Liang, Y. Liu, T.-M. Lok, and K. Huang, "Multiuser computation offloading and downloading for edge computing with virtualization," *IEEE Trans. Wireless Commun.*, vol. 18, no. 9, pp. 4298–4311, Sep. 2019.

[31] H. Zhang, Y. Qiu, X. Chu, K. Long, and V. C. M. Leung, "Fog radio access networks: Mobility management, interference mitigation, and resource optimization," *IEEE Wireless Commun.*, vol. 24, no. 6, pp. 120–127, Dec. 2017.

[32] Y.-J. Ku *et al.*, "5G radio access network design with the fog paradigm: Confluence of communications and computing," *IEEE Commun. Mag.*, vol. 55, no. 4, pp. 46–52, Apr. 2017.

[33] C. Lim, C. Ranaweera, E. Wong, and A. Nirmalathas, "Design and planning for fiber-based small cell backhauling," in *Proc. 20th Int. Conf. Transparent Opt. Netw. (ICTON)*, Bucharest, Romania, Jul. 2018, pp. 1–3.

[34] P. T. Dat, A. Kanno, N. Yamamoto, and T. Kawanishi, "Radio-over-fiber-based seamless fiber–wireless convergence for small cell and linear cell networks," in *Proc. Opt. Fiber Commun. Conf. (OFC)*, San Diego, CA, USA, 2018, pp. 1–3.

[35] W. Zhang, Y. Wen, K. Guan, D. Kilper, H. Luo, and D. Wu, "Energy-optimal mobile cloud computing under stochastic wireless channel," *IEEE Trans. Wireless Commun.*, vol. 12, no. 9, pp. 4569–4581, Sep. 2013.

[36] W. Tang and R. Jain, "Hierarchical auction mechanisms for network resource allocation," *IEEE J. Sel. Areas Commun.*, vol. 30, no. 11, pp. 2117–2125, Dec. 2012.

[37] M. M. Olama, S. M. Djouadi, and C. D. Charalambous, "Stochastic power control for time-varying long-term fading wireless networks," *EURASIP J. Adv. Signal Process.*, vol. 2006, no. 1, pp. 1–13, Dec. 2006.

[38] Y. Jiang, Y. Hu, M. Bennis, F.-C. Zheng, and X. You, "A mean field game-based distributed edge caching in fog radio access networks," *IEEE Trans. Commun.*, vol. 68, no. 3, pp. 1567–1580, Mar. 2020.

[39] B. K. Oksendal, *Stochastic Differential Equations: An Introduction with Applications*, 5th. Berlin, Germany: Springer-Verlag, 2010.

[40] P. Semasinghe and E. Hossain, "Downlink power control in self-organizing dense small cells underlaying macrocells: A mean field game," *IEEE Trans. Mobile Comput.*, vol. 15, no. 2, pp. 350–363, Feb. 2016.

[41] X. Ge, H. Jia, Y. Zhong, Y. Xiao, Y. Li, and B. Vucetic, "Energy efficient optimization of wireless-powered 5G full duplex cellular networks: A mean field game approach," *IEEE Trans. Green Commun. Netw.*, vol. 3, no. 2, pp. 455–467, Jun. 2019.

[42] M. De Mari and T. Quek, "Energy-efficient proactive scheduling in ultra dense networks," in *Proc. IEEE Int. Conf. Commun. (ICC)*, Paris, France, May 2017, pp. 1–6.

[43] M. Burger and J. M. Schulte, "Adjoint methods for Hamilton-Jacobi-Bellman equations," Dept. Math., Univ. Münster, Münster, Germany, Tech. Rep., 2010. [Online]. Available: https://www.semanticscholar.org/paper/Adjoint-Methods-for-Hamilton-Jacobi-Bellman-Burger-Schulte/58d9c7c79ca6b23687ae08fb84b8033ba3f0687e?sort=relevance&pdf=true

[44] M. Benaïm and J.-Y. L. Boudec, "A class of mean field interaction models for computer and communication systems," *Perform. Eval.*, vol. 65, nos. 11–12, pp. 823–838, Nov. 2008.

[45] Y. Achdou, F. Camilli, and I. Capuzzo-Dolcetta, "Mean field games: Numerical methods for the planning problem," *SIAM J. Control Optim.*, vol. 50, no. 1, pp. C77–C109, 2012.

[46] Y. Achdou and I. Capuzzo-Dolcetta, "Mean field games: Numerical methods," *SIAM J. Numer. Anal.*, vol. 48, no. 3, pp. 1136–1162, Jan. 2010.

[47] Y. Zhong, X. Ge, H. H. Yang, T. Han, and Q. Li, "Traffic matching in 5G ultra-dense networks," *IEEE Commun. Mag.*, vol. 56, no. 8, pp. 100–105, Aug. 2018.

[48] Y. Zhong, T. Q. S. Quek, and X. Ge, "Heterogeneous cellular networks with spatio-temporal traffic: Delay analysis and scheduling," *IEEE J. Sel. Areas Commun.*, vol. 35, no. 6, pp. 1373–1386, Jun. 2017.

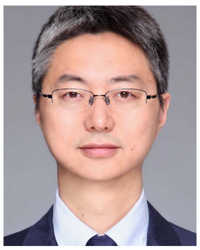
[49] L. Li *et al.*, "Resource allocation for NOMA-MEC systems in ultra-dense networks: A learning aided mean-field game approach," *IEEE Trans. Wireless Commun.*, vol. 20, no. 3, pp. 1487–1500, Mar. 2021, doi: 10.1109/TWC.2020.3033843.

[50] Y. Yu, J. Zhang, and K. B. Letaief, "Joint subcarrier and CPU time allocation for mobile edge computing," in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, Washington, DC, USA, Dec. 2016, pp. 1–6.

[51] F. Wang, J. Xu, X. Wang, and S. Cui, "Joint offloading and computing optimization in wireless powered mobile-edge computing systems," *IEEE Trans. Wireless Commun.*, vol. 17, no. 3, pp. 1784–1797, Mar. 2018.

[52] C. You, K. Huang, H. Chae, and B.-H. Kim, "Energy-efficient resource allocation for mobile-edge computation offloading," *IEEE Trans. Wireless Commun.*, vol. 16, no. 3, pp. 1397–1411, Mar. 2017.

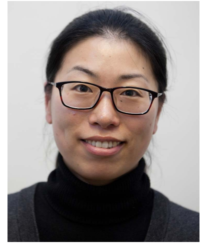
Renjun Zheng received the master's degree in communication and information system from Beijing Jiaotong University, Beijing, China. He is currently a Communication Standardization Engineer with China Aerospace Science and Technology Corporation, China. His research interests include satellite communications, 6G, and the IoT.



Haibo Wang (Member, IEEE) received the M.Sc.E.E. and Ph.D. degrees in wireless communications from Aalborg University, Denmark, in 2005 and 2008, respectively. He is currently the Associate Director of the Research Institute of Broadband Wireless Mobile Communications and an Associate Professor with Beijing Jiaotong University, Beijing, China. His research interests include (but not limited to) fog computing, device-to-device communications, mean field games, block chain, and system performance optimization. Besides research, he has also developed expertise in the design and development of mobile communication systems and networks while working with Nortel Networks, Ericsson, and Tieto.

Matthieu De Mari (Member, IEEE) received the Ph.D. degree from the Large Networks and Systems Group (LANEAS) in CentraleSupelec and CEA-Leti, in July 2015, under the supervision of Prof. M. Debbah and Prof. E. C. Strinati, on the topic "Radio Resource Management for Green Wireless Networks." From June 2016 to January 2020, he worked as a Postdoctoral Research Fellow with the Wireless Networks and Decision Systems (WDNS) group, Singapore University of Technology and Design, under the supervision of Prof. T. Quek. In March 2020, he joined the Singapore University of Technology and Design, as a Lecturer at the Information Systems Technology and Design (ISTD) Pillar.

Miao Cui (Member, IEEE) received the B.E. degree in communication engineering and the M.S. degree in computer science from Northeast Electric Power University, Jilin City, China, in 2001 and 2003, respectively, and the Ph.D. degree in circuit system from the South China University of Technology, Guangzhou, China, in 2009. She is currently a Lecturer with the Guangdong University of Technology, Guangzhou. Her research interests include the analysis, optimization, and design of wireless networks.



Xiaoli Chu (Senior Member, IEEE) received the B.Eng. degree in electronic and information engineering from Xi'an Jiaotong University in 2001 and the Ph.D. degree in electrical and electronic engineering from The Hong Kong University of Science and Technology in 2005. From 2005 to 2012, she was with the Centre for Telecommunications Research, King's College London. She is currently a Professor with the Department of Electronic and Electrical Engineering, The University of Sheffield, U.K. She has coauthored over 180 peer-reviewed journal articles and conference papers. She has coauthored/co-edited the books *Fog-Enabled Intelligent IoT Systems* (Springer 2020), *Ultra Dense Networks for 5G and Beyond* (Wiley 2019), *Heterogeneous Cellular Networks: Theory, Simulation and Deployment* (Cambridge University Press 2013), and *4G Femtocells: Resource Allocation and Interference Management* (Springer 2013). She was a co-recipient of the IEEE Communications Society 2017 Young Author Best Paper Award. She received the IEEE COMMUNICATIONS LETTERS Exemplary Editor Award in 2018. She is also a Senior Editor of the IEEE WIRELESS COMMUNICATIONS LETTERS and an Editor of the IEEE COMMUNICATIONS LETTERS.

Tony Q. S. Quek (Fellow, IEEE) received the B.E. and M.E. degrees in electrical and electronics engineering from the Tokyo Institute of Technology in 1998 and 2000, respectively, and the Ph.D. degree in electrical engineering and computer science from the Massachusetts Institute of Technology in 2008.

He is currently the Cheng Tsang Man Chair Professor with the Singapore University of Technology and Design (SUTD). He also serves as the Director for the Future Communications Programme, the Head for ISTD Pillar, and the Deputy Director for SUTD-ZJU IDEA. His current research interests include wireless communications and networking, network intelligence, the Internet-of-Things, URLLC, and big data processing. He was honored with the 2008 Philip Yeo Prize for Outstanding Achievement in Research, the 2012 IEEE William R. Bennett Prize, the 2015 SUTD Outstanding Education Awards–Excellence in Research, the 2016 IEEE Signal Processing Society Young Author Best Paper Award, the 2017 CTTC Early Achievement Award, the 2017 IEEE ComSoc AP Outstanding Paper Award, the 2020 IEEE Communications Society Young Author Best Paper Award, the 2020 IEEE Stephen O. Rice Prize, the 2020 Nokia Visiting Professor, and the 2016–2020 Clarivate Analytics Highly Cited Researcher. He has been actively involved in organizing and chairing sessions, and has served as a member of the Technical Program Committee and a symposium chair in a number of international conferences. He is also serving as an Editor for the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS and an Elected Member for the IEEE Signal Processing Society SPCOM Technical Committee. He was an Executive Editorial Committee Member of the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, and an Editor of the IEEE TRANSACTIONS ON COMMUNICATIONS and the IEEE WIRELESS COMMUNICATIONS LETTERS. He is also a Distinguished Lecturer of the IEEE Communications Society.